Thermal Lowering of the Threshold for Microwave Breakdown in Air-Filled Waveguides

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Abstract —**It is demonstrated that the presence of absorbing inhomogeneities in an air-filled microwave waveguide may significantly lower the threshold for microwave breakdown. The underlying physical mechanism is locally enhanced microwave absorption and subsequent heating of the waveguide air. The results provide an explanation of the fact that breakdown thresholds of microwave transmission systems are observed to be much lower than theoretical predictions.**

I. INTRODUCTION

THE MICROWAVE breakdown strength of air at atmospheric pressures has been theoretically determined to be $E/p \simeq 30$ V/cm·torr for CW operation, where E is the rms strength of the microwave electric field and p is the air pressure (see e.g. [1]). This result does agree very well with microwave cavity experiments performed under well-controlled conditions [2]. However, the breakdown strength of "realistic" microwave waveguide systems may be up to an order of magnitude lower [3], [4]. This implies a practical power breakdown threshold which may be only a few percent of the theoretical value.

In a recent article [5], the power-handling capability of dielectric waveguides at millimeter wavelengths was discussed. It was emphasized that although the breakdown strength of a dielectric material may be very high, dielectric heating can cause the waveguide material (polystyrene or PTFE) to melt at comparatively low power levels. Thus, the maximum allowable power level in the waveguide is set by dielectric heating rather than by dielectric breakdown

An analogous situation can occur in connection with microwave propagation in air-filled waveguides. In the presence of lossy foreign material in the waveguide, or if local electric field enhancement creates regions with high ionization, the temperature of the air in the waveguide is increased due to the enhanced microwave power absorption. As we will show in the present investigation, this may lead to a significant lowering of the breakdown strength.

The theoretical analysis is based on a model where the microwave power absorption is caused by a finite electron density. We then determine the resulting stationary maximum temperature of the waveguide air from the heat balance between microwave absorption and heat loss due to thermal conduction. Breakdown occurs when the microwave electric field is equal to the temperature-reduced breakdown strength. This gives a self-consistent breakdown level, which can be significantly less than the conventionally cited breakdown strength for air.

In a further step, we pursue this effect by investigating how small strongly absorbing regions can cause full-scale breakdown by initiating a thermal expansion wave. It is found that this effect can easily cause microwave breakdown at power levels even less than those associated with full-scale temperature effects.

II. HEAT BALANCE EQUATION

The evolution of the temperature T of the waveguide air is determined by the heat conduction equation:

$$\rho C \frac{\partial T}{\partial t} = \nabla(\kappa \nabla T) + W \tag{1}$$

where ρ is the density, $C(T)$ is the specific heat, $\kappa = \kappa(T)$ is the thermal conductivity of air, and W is the absorbed microwave power density. For temperatures $T \leqslant 2000$ K, we approximate the temperature dependence of κ as

$$\kappa(T) \simeq 6.15 \cdot 10^{-4} \tilde{T}^{3/4} \left(\frac{\text{W}}{\text{cm K}}\right) \tag{2}$$

where the tilde denotes that T is measured in units of 1000 K [6].

By introducing the heat flow potential θ according to [7]

$$\theta = \int_0^T \kappa(T')\, dT' \tag{3}$$

we can write the thermal conduction term as $\nabla(\kappa \nabla T) = \nabla^2 \theta$, where

$$\theta \simeq 0.35 \tilde{T}^{7/4} \left(\frac{\text{W}}{\text{cm}}\right). \tag{4}$$

The absorbed microwave power density W is determined by the absorption coefficient α according to

$$W = 2\alpha k_0 P \tag{5}$$

where P is the intensity (W/cm^2) and k_0 is the wavenumber of the microwave.

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We consider situations where the absorption is small, i.e., $\alpha \ll 1$. This implies the following approximation for α:

$$\alpha \simeq \frac{1}{2}\frac{\omega}{\nu}\frac{n}{n_c} \tag{6}$$

where ω is the angular wave frequency, n is the free electron density in the waveguide air, ν is the electron collision frequency, and n_c is the critical electron density at which the wave frequency ω equals the plasma frequency ω_p.

The collision frequency ν will be approximated by its electric field independent value [1]

$$\nu/p \simeq 5\cdot 10^9 \left(\frac{1}{\text{s}\cdot\text{torr}}\right) \tag{7}$$

which is valid for electric fields that are not too small.

In the present context, which includes temperature effects, it is crucial to note that the important physical parameter determining the collision frequency is not the air pressure p but rather the density of neutral particles N [8]. This fact can be accounted for by using instead of p the reduced pressure p^* [9], which for constant pressure at room temperature is defined by

$$p^* = p\frac{300}{T}. \tag{8}$$

Collecting the expressions (5)–(8) and taking $p = 760$ torr, we obtain

$$W \simeq \frac{\tilde{T}}{\lambda_0^2}\frac{n}{n_c}P \tag{9}$$

where λ_0 is the wavelength of the microwave in cm.

In steady state and expressed in terms of the heat flow potential θ, (1) becomes

$$\nabla^2\theta + A\theta^{4/7} = 0 \tag{10}$$

where

$$A = 1.8\frac{n}{n_c}\frac{P}{\lambda_0^2}. \tag{11}$$

For the present purpose, it is sufficient to solve (10) in a one-dimensional plane configuration together with the boundary conditions

$$\frac{d\theta}{dx}(0) = 0$$

$$\theta(\pm L) = \theta_W \tag{12}$$

where $2L$ corresponds to the smallest transverse dimension of the waveguide and θ_W is the value of the heat flow potential at the waveguide wall.

We are particularly interested in the maximum value of θ, i.e., $\theta_m = \theta(0)$, which can easily be found by integrating (10) twice to obtain

$$\theta_m^{-3/14}L\left(\frac{14A}{11}\right)^{1/2} = \int_{\theta_W/\theta_m}^{1}\frac{dt}{\sqrt{1-t^{11/7}}}. \tag{13}$$

Assuming $\theta_W/\theta_m \ll 1$, the integral in (13) can be evaluated approximately using the fact that $11/7 \simeq 3/2$ to give the following expression for the maximum temperature in the waveguide $\tilde{T}_m$:

$$\tilde{T}_m \simeq 1.3\left(\frac{n}{n_c}\frac{L^2}{\lambda_0^2}P\right)^{4/3} \tag{14}$$

where P is expressed in W/cm^2.

III. Thermal Breakdown

The threshold for attachment controlled CW microwave breakdown in air at room temperatures is

$$E/p \simeq 30\left(\frac{\text{V}}{\text{cm}\cdot\text{torr}}\right) \tag{15}$$

where E is the effective rms electric field [1], [2]. This implies a power threshold

$$P = P_0 \simeq 1.5\cdot 10^6\ (\text{W/cm}^2). \tag{16}$$

However, in temperature-dependent situations the pressure used in (15) should be replaced by the reduced pressure p^*. This results in a temperature-dependent breakdown power P_b, which for constant pressure p is determined by

$$P_b = P_0\left(\frac{T_0}{T}\right)^2. \tag{17}$$

Thus, even if the incident wave power P is below the cold breakdown threshold ($P < P_0$), the temperature increase given by (14) may cause thermal breakdown if $P \geqslant P_b$. The new self-consistent breakdown strength is determined from the condition

$$\tilde{T} = \tilde{T}_0\left(\frac{P_0}{P_b}\right)^{1/2} = \tilde{T}_m \simeq 1.3\left(\frac{n}{n_c}\frac{L^2}{\lambda_0^2}P_b\right)^{4/3} \tag{18}$$

i.e.,

$$P_b \simeq 22\left(\frac{n_c}{n}\frac{\lambda_0^2}{L^2}\right)^{8/11} \tag{19}$$

Equation (19) reveals that the parameters determining the breakdown threshold are n/n_c and L/λ_0. The physical background of this scaling is the balance between microwave absorption ($\sim n/n_c$) and thermal conduction loss ($\sim \lambda_0^2/L^2$). Thus, for $f = 10$ GHz, we find $n_c \simeq 1.2\cdot 10^{10}$ cm^{-3}, which, together with $n/n_c = 10^{-4}$, implies $n \simeq 10^8$ cm^{-3}. This corresponds to a degree of ionization which is very small, but nevertheless much larger than thermal equilibrium values at room temperatures.

Consequently, in a perfect undisturbed waveguide, the absorption (as determined by n/n_c) is too small to be of any significance with respect to thermal heating. However, if some mechanism happens to create, somewhere in the waveguide, a preionized region with enhanced electron density, thermal breakdown can occur at much lower values of microwave power than predicted by cold-air theory.

Such high-electron-density regions can occur, e.g, when local electric field enhancement creates minor discharges or when lossy foreign material [3] is present in the waveguide.

This is similar to the optical case where cracks and micropores in the dielectric may lead to concentrations of the electric field strength and to a subsequent lowering of the nominal external intensity for avalanche breakdown [10].

IV. Breakdown Due to Heat Wave Expansion

Although the analysis of the previous sections clearly shows that strongly absorbing regions of dimensions comparable to the waveguide size are potentially dangerous with respect to microwave breakdown, an even more serious limitation may be set by absorbing inhomogeneities of size much less than the wavelength of the microwave. This is a well-known phenomenon in connection with optical breakdown in transparent dielectrics [11], but as far as we know its significance for microwave breakdown has not been analyzed in detail.

The physical picture of the initiation of the breakdown process is as follows. When a small strongly absorbing inhomogeneity is heated, absorption in the surrounding medium also increases, resulting iin an increased effective size of the inhomogeneity. Under certain conditions, a combined heat and absorption wave may propagate outward, leading to fully developed breakdown.

The theory for this thermal instability in optical transparent media was first given in [11]. We will here present the main steps in a modified version of that analysis, adapted to apply to microwave breakdown in air. We consider a small strongly absorbing region of radius R, which transfers the fraction βP of the absorbed power to the surrounding air by thermal conduction.

In steady state, the temperature in the surrounding air ($r \geqslant R$) is determined by the equation

$$\nabla(\kappa \nabla T)+2\alpha k_0 P=0 \tag{20}$$

together with the following boundary condition for the heat flow at $r=R$:

$$|-\kappa \nabla T|=\beta P, \qquad r=R. \tag{21}$$

The strong absorption in the small sphere will result in very high temperatures, which also implies high temperatures in the surrounding air closest to the absorbing region. For temperatures $T \geqslant 2000$ K, the ionization properties of air are determined by the presence of nitrogen oxide [7], with an ionization energy $E \simeq 9.25$ eV. The electron density is then determined by the Boltzman–Saha relation according to $n=n_0\tilde{T}^{1/4}\exp(-E/2T)$, where $n_0 \simeq 2.7 \cdot 10^{18}$ cm^{-3} [7]. This implies that the temperature dependence of the microwave absorption coefficient can be written

$$\bar{\alpha} \equiv 2\alpha k_0 = a\tilde{T}^{5/4}\exp\left(-\frac{E}{2T}\right) \tag{22}$$

where $a=n_0/(n_c\lambda_0^2)$.

In view of the strong temperature dependence of α and in order to simplify our analysis, we will neglect the temperature variation of κ and take $\kappa \simeq \kappa(2000 \text{ K}) \simeq 1$ W/(cm·1000 K).

Furthermore, using the fact that the most important factor determining the temperature in the region $r \geqslant R$ is the heat flow from the strongly absorbing region ($r \leqslant R$), cf. (21), the solution of (20) and (21), $T_0(r)$, is given by

$$T_0(r) \simeq \frac{\beta P}{\kappa}\frac{R^2}{r}. \tag{23}$$

The stability of this solution is determined by the linearized equation for a small temperature disturbance, $T_1(t,r)$, viz.

$$\rho C\frac{\partial T_1}{\partial t}=\kappa\nabla^2 T_1+\frac{\partial\bar{\alpha}}{\partial T}(T_0(r))PT_1. \tag{24}$$

The boundary conditions on (24) are $T_1(r,t) \to 0$ as $r \to \infty$ for every t and $\partial T_1(R,t)/\partial r=0$.

The solution of (26) is represented in the form

$$T_1(r,t)=\sum_\gamma C_\gamma^{-\gamma t}\phi_\gamma(r) \tag{25}$$

which implies the following equation for ϕ_γ:

$$\nabla^2\phi_\gamma+(\bar{\gamma}+V(r))\phi_\gamma=0 \tag{26}$$

where $\bar{\gamma}=\gamma C\rho/\kappa$ and $V(r)=(P/\kappa)\partial\bar{\alpha}(T_0(r))/\partial T$.

Equation (26) is a Schrödinger equation with a potential $V(r)$ which falls off very rapidly for $r \geqslant R$ due to the strong temperature dependence of $\bar{\alpha}(T)$. The problem is then that of finding a bound state in a deep narrow well. Following standard procedures [11], the instability condition (sign $\bar{\gamma}<0$) becomes

$$\int_R^\infty V(r)\,dr>\frac{1}{R}. \tag{27}$$

Using (21) and the definition of $V(r)$, this can be rewritten as

$$R\bar{\alpha}(T_0(R))>\beta$$

or explicitly

$$P>P_b \equiv \frac{E\lambda}{\beta R}\left[\ln\frac{a(R)R}{\beta}\tilde{T}_0^{5/4}(R)\right]^{-1}. \tag{28}$$

The weak functional dependence of the logarithm in (28) allows further approximations. The dominating factor in the argument is $a=n_0/(n_c\lambda_0^2) \approx 2.5\cdot 10^5$. As typical values to be used in the logarithm we take $\tilde{T}_0 \simeq 2$, $\beta \simeq 0.1$, and $R \simeq 10^{-2}$ cm, which implies the following simple expression for the breakdown threshold P_b:

$$P_b \simeq \frac{4}{\beta R}\left(\frac{\text{W}}{\text{cm}^2}\right). \tag{29}$$

For a strongly absorbing inhomogeneity, we can as an example take $\beta \simeq 0.1$ and $R=2\cdot 10^{-2}$ cm, which yields $P_b \simeq 2$ kW/cm^2, i.e., an order of magnitude lower than the previously found threshold due to large-scale thermal breakdown.

V. Concluding Remarks

Several experiments exist which, at least qualitatively, confirm the present results. Reference [3] investigates how the presence of foreign particles of different materials affects the breakdown threshold. It was observed that breakdown was initiated only when the particles were sufficiently lossy to allow heating to orange or white temperatures. The existence of a critical temperature was especially emphasized. Particles heated to a dull red temperature could remain in the waveguide for long periods of time without causing breakdown. From (23), we have $T_0(R) \simeq \beta PR/\lambda$, which implies that the observed temperature is a measure of the absorption β. Thus, the observations confirm the scaling of P_b with β. It is also interesting to note that the observed breakdown threshold was typically a few kW/cm^2.

Furthermore, [12] investigates the change in breakdown threshold of a microwave cavity when a small hemispherical conducting boss was inserted on one of the cavity walls. It was found that the breakdown threshold was significantly lowered from the value $E/p = 30$ V/(cm·torr) when the boss was introduced. In addition, the reduction of the breakdown strength increased with the radius of the boss, which was typically of the order $(2–4)\times 10^{-2}$ cm. Thus, this observation confirms the scaling of P_b with R.

The results of the present study clearly demonstrate that local regions of enhanced microwave absorption in a waveguide system could develop into full-scale breakdown discharges, even in situations where the electric field strength is well below the theoretical breakdown threshold of cold air. The results also emphasize the importance of avoiding waveguide design features which give rise to local enhancement of the electric field as well as those which introduce foreign lossy material into the waveguide.

References

[1] W. Woo and J. S. DeGroot, "Microwave absorption and plasma heating due to microwave breakdown in the atmosphere," *Phys. Fluids*, vol. 27, p. 475, 1984.

[2] A. D. McDonald, *Microwave Breakdown in Gases*. New York: Wiley, 1966.

[3] W. Beust and W. W. Ford, "Arcing in CW transmitters," *Microwave J.*, vol. 10, p. 91, 1961.

[4] Ya. Brodsky, S. V. Goluber, V. G. Zorin, A. G. Luchimin, V. E. Semyonov, and G. M. Fraiman, "Dynamics of a HF discharge in an oversized waveguide," in *Proc. 15th Int. Conf. Ionized Gases* Minsk (USSR), 1981, p. 121.

[5] D. G. Jablonski, "Power-handling capabilities of circular dielectric waveguide at millimeter wavelengths," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, p. 85, 1985.

[6] W. E. Forsythe, *Smithsonian Physical Tables*. Baltimore, MD: Smithsonian Institution, 1954, p. 743.

[7] Yu. P. Raizer, "Propagation of a high-pressure microwave discharge," *Soviet Phys.—JETP*, vol. 34, p. 114, 1972.

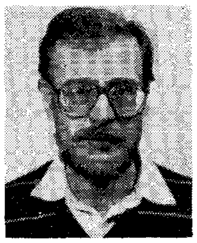
[8] E. Ya. Kogan and B. Yu. Kuzin, "Steady-state ionization waves in subthreshold rf field," *Sov. J. Plasma Phys.* vol. 11, p. 357, 1985.

[9] G. C. Light and E. C. Taylor, "Microwave breakdown in high-temperature air," *J. Appl. Phys.*, vol. 39, p. 1591, 1968.

[10] N. Bloembergen, "Role of cracks, pores, and absorbing inclusions on laser induced damage threshold at surfaces of transparent dielectrics," *Appl. Opt.*, vol. 12, p. 661, 1973.

[11] S. I. Anisimov and B. I. Makshantsev, "Role of absorbing inclusions in the optical breakdown of transparent media," *Sov. Phys.—Solid State*, vol. 15, p. 743, 1973.

[12] P. M. Platzman and E. H. Solt, "Microwave breakdown of air in nonuniform electric fields," *Phys. Rev.*, vol. 119, p. 1143, 1960.



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